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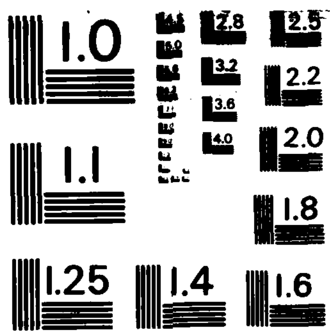

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FINAL REPORT

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MULTIPLE SCATTERING OF LIGHT IN SEAWATER

TETRA TECH

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FINAL REPORT

MULTIPLE SCATTERING OF LIGHT IN SEAWATER

Light scattering is the basic physical limitation in certain naval operations. These include:

- Optical detection of submarines,
- Blue-green communications to submarines,
- Undersea visibility in search, rescue, and mapping.

To evaluate these limitations, the Navy needs solutions to the mathematical problem of multiple scattering, or radiative transfer as physicists prefer to call it. To describe this problem, we need to make a distinction between inherent and apparent properties of the seawater. The inherent properties refer to the water and the particles suspended therein:

α — the beam attenuation coefficient,

$\sigma(\psi)$ — the volume scattering function,

a — the absorption coefficient,

S_n — moments of ϕ (delta) (spherical harmonics),

s — total scattering coefficient; ~~$s = S_0 = \int \sigma d\omega$~~

A_n — ϕ (alpha) $- S_n$ (sub n).

These quantities are not independent, for example, $\alpha = a + s$ and $A_0 = a$, $A_\infty = \alpha$.

The apparent properties are simply the radiance field $L(z,\theta,\phi,t)$ and various integral moments thereof. We use the spherical moments

$$L_n^m = \int P_n^m L(\theta,\phi) \cos m\phi \; d\omega$$

These depend not only on the inherent properties of seawater, but also on sun angle (or other light source), sea state, depth, type of bottom and so on.

The usual, classic, or "direct" problem of radiative transfer is

inherent $\Longrightarrow$ apparent,

i.e. find the radiance field given the geometry of illumination and inherent properties of seawater. The "inverse" problem of radiative transfer is

apparent $\Longrightarrow$ inherent.

The naval applications mentioned above always require solutions to the direct problem. However, the inherent properties that go into the solution have not been well surveyed throughout the world, nor completely as a function of depth, and so the Navy also needs solutions to the inverse problem as a means for designing instruments to make these surveys.

Under the contract N00014-81-C-0288, we have made contributions to both the direct and inverse problems. The main effort has been on the direct problem because it is mathematically difficult and besides, we at least know some typical values of the inherent properties to put into the problem. The inverse problem is mathematically rather simple, although many people do not recognize this simply because they have not focussed attention on the particular formulation that makes it simple.

Sections I & II

Section ~~I below~~ describe~~s~~ ~~our~~ our contributions to the direct and inverse problem~~s~~ of radiative transfer. Section II summarizes the inverse, and Sec. III lists the papers and reports prepared and published under the subject contract.

1.0 DIRECT PROBLEM OF RADIATIVE TRANSFER

The following subsections describe ways of treating the direct problem and point out their strengths and weaknesses. Further subsections describe our breakthrough and the papers that have resulted from it.

1.1 Small Angle Scattering

Some of the problems that require a theory of multiple scattering may be called imaging problems, if only in a rather crude sense in which the object of interest appears as an extended blur that subtends 30 to 50°. As long as most of the rays that form the image have scattered through angles less than 30° in the water (not counting surface roughness), then the radiative transfer problem can be treated in the small-angle approximation. (Diffuse light is still of some interest since it causes background radiance that reduces contrast, but the accuracy need not be nearly as great as the details of the image.) The small-angle approximation greatly relieves the computational difficulties and allows us to treat cases with irregular objects and boundaries. We treated this subject in 1969 as reported in the Journal of the Optical Society (Vol. 59, pp. 686-691, June 69). We extended the subject in several directions as described in AGARD Lecture Series #61, Sept. 73. (In fact, this lead into the spherical harmonic work reported here.) In brief, we take a Fourier transform of the image that would be seen without any scattering particles in the intervening medium (seawater, fog, whatever). Then we apply an attenuation factor to each Fourier component, a factor that integrates certain scattering properties of the medium along the rays. Finally, we reconstitute the image from the attenuated transform. (Note that spherical moments become Fourier transforms in the limit of high harmonics.)

1.2 Photon Tracking

In nature, photons are absorbed or scattered at random. The computer codes for tracking them merely simulate the process

directly by tracking individual photons. The Monte Carlo types generate random numbers that determine their fate. (The computer first generates random numbers from a uniform distribution, then takes a function of these numbers to get numbers properly distributed for free path lengths and scattering angles.) Other codes integrate over the probability distributions for several scattering events without having random numbers. By far the strongest point in favor of photon-tracking is its generality. It applies in any geometry with boundaries and sources of irregular shape. The greatest drawbacks are lack of insight and transferability. When you run a case, you get one number with no further insight into the problem and then wonder if the results are correct. Cases take a long time to run because they have to trace the history of a great many photons before they converge to an answer. Monte Carlo takes longer to converge because the statical noise has to average out over many cases. Some codes can be much less random than others by integrating over the probability distribution of different scattering angles instead of sampling the distribution at random. Running time is especially long for cases deep in the scattering medium where typical rays have scattered many times. In summary, photon-tracking codes have brute-force generality, which makes them ideal for shallow problems with irregular shapes.

1.3 Analytic Methods

These are methods of solving the integro-differential equation of radiative transfer, usually in the form of the Boltzmann equation. This is essentially an equation of continuity that keeps track of all the radiance that has not been absorbed. Of all the basic equations in engineering and applied physics, Boltzmann's is one of the most resistant to general solutions because it contains an arbitrary function (the volume scattering function), partial derivatives, and an integral. For many decades, researchers have devised approximate solutions. But these solutions either have been crude or have applied to cases in which the scattering is nearly isotropic. In particular, the big developments in nuclear engineering required solutions to keep track of neutrons,

but the initial assumption in all this work is that the scattering is either isotropic or nearly so. By contrast, typical scattering in seawater (and clouds) is very highly anisotropic and changes with depth as well, and so these are the cases we have considered exclusively.

Some methods for treating this class are very crude and yield only the grossest features such as up- and downwelling currents of radiation. We are concerned here with the more refined methods that give details of the radiance distribution. The main advantage over photon-tracking is speed of computation and physical insight that accompanies the formulas. Analytic methods are especially appropriate for deep cases, because the answer comes in the form of eigenfunctions of radiance that retain their shape as they decay with distance. In deep cases, only one or two eigenfunctions survive, the others having decayed to negligible levels. All other eigenfunctions are merely surface transients near the boundary of that medium.

The main drawback of anlytic methods is the same as that of any boundary value problem defined by a differential equation: only symmetrical cases, i.e. plane, spherical, or cylindrical boundaries with point sources, linear sources, or infinite beams, can be solved with precision. Extending these requires various perturbation theories or other approximations. Fortunately, most cases of interest do have the simple boundaries. The ocean has a plane surface. (Except in the most extreme cases, the radiative transfer problem begins underneath the surface roughness, and refraction at the surface can be treated separately.) Usually the bottom is either far enough to treat as infinite or smooth enough to consider planar. Most of the stratification is planar. Sunlight is treated as a beam of infinite extent, so is a wide beam such as that from a high-altitude aircraft or satellite. A wake is an infinite thin line. A lamp used in measurements is a spherical source. Only occasionally do we have unfortunately mixed geometry, for example a lamp (spherical) near the surface (planar).

1.4 Tetra Tech Method

We have made a significant breakthrough in analytic techniques using spherical harmonics and have published a lengthy (35 pages) paper on the subject entitled, "Anisotropic Multiple Scattering of Collimated Irradiance", which we refer to as WS (for the authors, Wells and Sidorowich). It appears in the Annals of Physics, Vol. 144, pg. 203 (Dec 82). The editors of this journal are all professors of physics at MIT and Harvard. The following excerpt from the introduction to WS explains the general nature of our innovations:

> Spherical harmonics are frequently used to solve the Boltzmann equation for radiative transfer because the method is capable of arbitrarily high (or low) accuracy according to the number of harmonics used. The method is not usually considered suitable for a medium having a very pointed scattering function because it leads to hundreds, or even thousands, of coupled differential equations for harmonic coefficients of high degree. The theory is so cumbersome because it is capable of treating the most general boundary conditions, e.g. N beams incident from N directions simultaneously. We simplify the equations in a way that limits them to one direction of incidence at a time, and if anyone needs two or more beams, he may superpose solutions since the equations are linear. We show that the equations can be uncoupled for high harmonics (our examples use $n > 8$) while the coupled set for low harmonics can be terminated in a way that mates naturally with the highs.
>
> We demonstrate the method using slab geometry: an infinite beam of particles or light irradiates a scattering medium with a plane boundary. At the transition from low to high harmonics, we rotate the z-axis of the spherical coordinates system from the direction perpendicular to the boundary to the direction pointed at the source. The advantage is best explained with a familiar example, the appearance of the full moon seen through a haze that produces a glow field or halo of small-angle scatter. The spherical harmonics of lowest degree describe the wide-angle diffuse radiance that intercepts the horizon, and so the natural coordinate system uses zenith angle θ with the horizon at 90°. However, the harmonics of high degree describe the peak glow from directions near the moon, which for all practical purposes are symmetric about it. By rotating to coordinates pointed at the moon, we require only the axi-symmetric harmonics $P_n(\cos\theta)$ and not the more general ones $P_n^m(\cos\theta)\cos m\phi$. To describe the radiance with resolution of 1°, we need over 200 symmetric harmonics. But if we use normal coordinates, we need >20,000 of them.

Figure 1 shows the dimensions and parameters in the problem. Here α and σ denote the inherent properties of the medium. Other parameters are time and geometrical dimensions. The apparent angle of incidence is ζ (after refraction at the surface). The position of the observer is x, y, z. The direction of observation is θ, ϕ. The most general problem is to find $L(\theta,\phi;x,y,z,t;\zeta)$. As you might expect, this is too difficult. However, we have solved cases with four arguments quite handily, especially $L(\theta,\phi;z;\zeta)$, and also a few cases of $L(\theta;z,t;0)$. In the former case, the source is independent of time and infinite in the x and y directions (like sunlight under a clear sky). In the latter case, the source is time dependent at normal incidence and infinite in extent. We have also treated cases in which the parameters of the medium vary with depth z. Strong variations in x and y, caused either by variation in the source or the medium, are particularly difficult generalizations. We have worked on the case of spherical radiance surrounding an isotropic point source and have encountered curious difficulties. However, we have not exhausted schemes to finish the job and find $L(\theta,r)$, Fig. 2. Overall, we have developed a good intuition for choosing methods that give reasonable accuracy in a wide variety of cases. In all cases, radiance is expanded in spherical harmonics, L_n^m, and the harmonics of high degree (greater than 3 or 7) are split off and expressed as simple exponential decays for each order. In one important case

$$L_n = \exp(-A_n z \sec\zeta), \tag{1}$$

where A_n is readily derived from spherical harmonics of the scattering function.

A great advantage is that the expressions have intuitive interpretation relating to paths from the boundary to the sensing radiometer, or optical receiver. For example, the expression $z \sec\zeta$ in Eq. 1 is just the slant path length from the surface to the sensor. This intuitive guidance lets us immediately write predictive formulas for more complicated cases. For example, a stratified ocean gives A_n as a function of z and Eq. 1 becomes

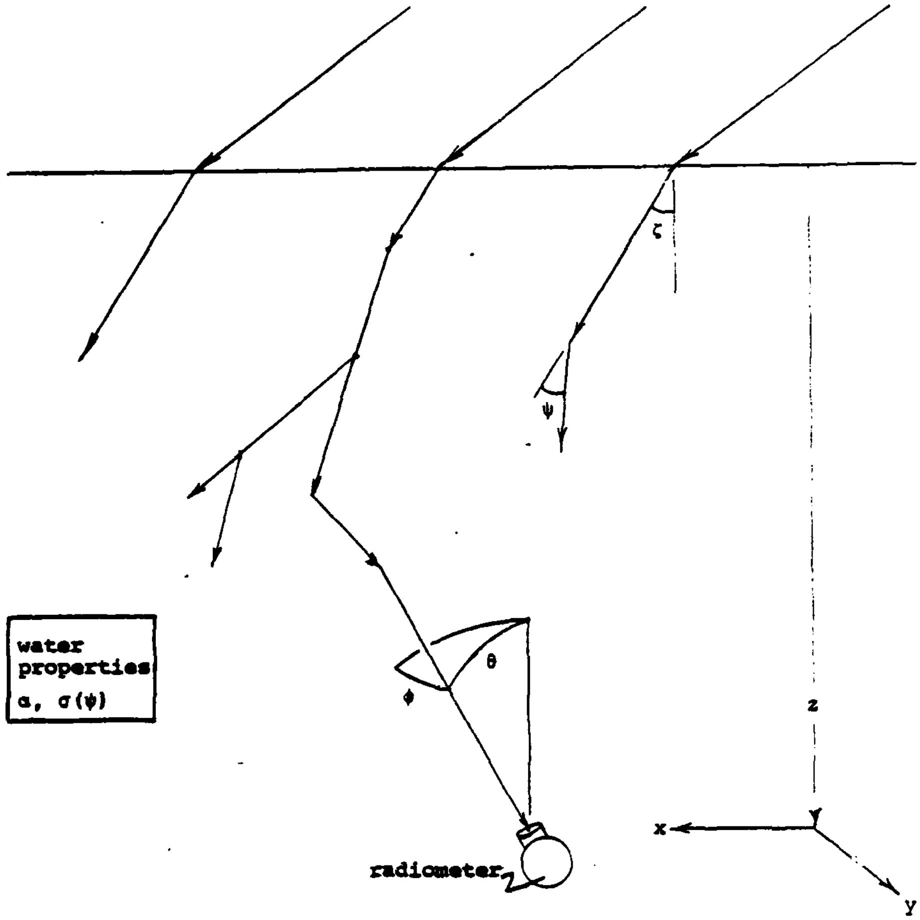

Fig. 1. Geometry of Radiance in Planar Case

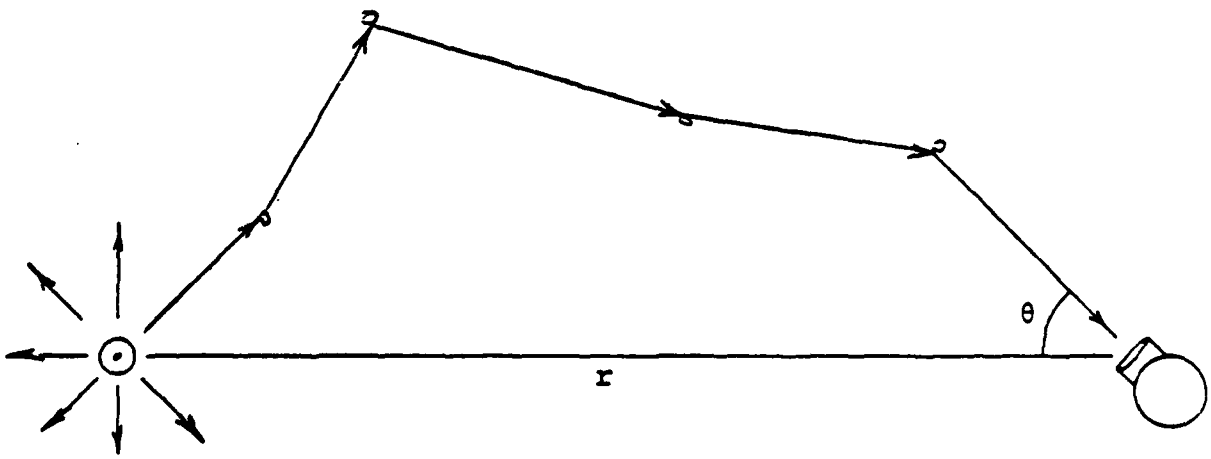

Fig. 2. Geometry of Radiance in Spherical Case

$$L_n = \exp\left[-\sec\zeta \quad \int_0^z A_n(z')dz'\right]$$

Similarly, cloud shadows, finite beams, etc. can all be managed by assigning appropriate power to various paths. (This is roughly analogous to solving optical diffraction patterns by a Fresnel integral over the aperture, which is much easier and more intuitive than solving Maxwell's equation with the aperture as a boundary condition. These considerations can be extended to other geometrics.

A second advantage of this approach is clarified by analogy to tracing the pulses of data through an electric communication link. One usually expands the signal in sinusoids and inquires about the attenuation of a certain required frequency. By analogy, a spherical moment of radiance represents a certain level of angular detail (instead of temporal) required in an application. Then Eq. 1 immediately gives the attenuation of that particular moment without requiring a complete solution for the radiance everywhere.

1.5 Recent Extensions

As an outgrowth of WS, we have prepared two more papers and submitted them for publication. These are the ones called A and B in our proposal P-26071 of August 82.

1.5.1 Variable Medium

The first extends the method of WS to variable media. In this computation, we find radiance as a function of depth and look angle in a scattering medium whose properties vary with depth, as they do in the ocean. That is, the total attenuation constant α becomes a function of depth, $\alpha(z)$, and so does the scattering function $\sigma(z,\psi)$. In terms of spherical harmonics, their decay constants A_n become functions $A_n(z)$. In an example reasonably representative of the real ocean, we treated a half space in which $A_n(z)$ varies over the range $0 < z < Z$ representing the biological surface layer, but A_n is constant for $z > Z$ representing deep water.

This paper has been accepted by the Journal of Computational Physics.

1.5.2 Computational Techniques

This paper is being submitted now to the Journal of Quantitative Spectroscopy and Radiative Transfer. Preprints for ONR are being mailed simultaneously. The paper gives a number of computational techniques that improve the solutions in WS. One of these (Section III) is called "extra integration" (B in our proposal). It is particularly effective in improving the solution close to boundaries. Figure 4 in this paper shows close agreement between our results and those of a computation by Devaux & Siewert [J. Appl. Math. & Phys. (ZAMP) 31, 592 (1980)] who use a very accurate numerical method.

Section V in our new paper also gives analytic means for improving the convergence of the infinite sum of spherical harmonics. This occurs in the conversion from moment of radiance L_n^m to radiance as a function of angle, $L(z,\theta,\phi)$. Other sections aid in the selection of an optimum approximation to match the direction of incidence, and give rules for optimizing the approximation in cases where the direction of incidence is blurred. (In the case of seawater, this applies to the roughness of the surface.)

Finally, Sec. IV of the new paper discusses the fact that α, the beam attenuation coefficient, is rather poorly defined. (It depends on resolution of the transmissometer.) Both the philosophical implications and the practical computational results are discussed.

2.0 INVERSE PROBLEM AND METERS FOR SEAWATER

Instruments have been devised to measure inherent properties α and $\sigma(\psi)$ directly. Then s and a may be deduced as $s = \int \sigma d\omega$, $a = \alpha - s$. For maximum control, these meters use internal lamps and function independently of sunlight. To the extent that these instruments are adequate, the Navy can get along without solving the inverse problem. However, a problem with them is that dimensions of the meters are limited to wieldy sizes of a meter or so. In this distance, one may not obtain a statistical average of the larger objects in the sea. Moreover, in some cases the effects are so small across one meter that inherent properties are not measured with enough precision to extend results to long paths on the order of a 100 meters in some practical applications.

To avoid these problems, the inherent properties may be deduced from measured radiance, but only indirectly through the equations for radiative transfer and then only with a loss of information. For example, one cannot deduce the small-angle scattering function in deep water from sunlight measured there because the light is already thoroughly diffused when it reaches this depth, and so small-angle scatters do not change the distribution appreciably.

This loss of information can be corrected by putting lamps somewhere near the radiometer. Unlike the sunlit case, the lamps are close enough to permit measurement of all scattering angles of interest. Unlike the self-contained instruments, the lamps are external and distant enough to register a large effect. However, the distance that averages over a large volume and gives the large effect also permits a significant amount of multiple scattering, and so again one must use the equations of radiative transfer to deduce the inherent properties from the radiometric data.

We have devised a new radiometer design that simplifies this deduction for oceanographic instrumentation. Both the theory of the radiometer and preliminary optical design are described in our paper in Applied Optics entitled, "Techniques for Measuring Radiance in the Sea and Air".

Optical oceanographers are accustomed to the planar geometry of collimated sunlight incident on a plane surface. But the rays from a lamp spread spherically. It is natural to assume that you merely factor out the inverse square law and treat the problem as though it were otherwise the same; but this happens to be wrong. Figure 3 shows that a single-scatter event with angle ψ in planar geometry contributes to the observed radiance at angle $\theta = \psi$ (measured from the source direction to the radiometer's axis). But a similar event in spherical geometry (same figure) leads to observed radiance at a far smaller angle. Moreover, in the planar case, the radiance reaches an asymptotic angular distribution $F(\theta,\phi)$ that decays exponentially, as mentioned earlier, i.e. $L\ F(\theta,\phi)\exp(-kz)$. In the spherical case, F and k are the same, but $L \sim r^{-1}F(\theta,\phi)\exp(-kz)$, not r^{-2} as you might think. Section VI in the <u>Applied Optics</u> paper makes this distinction and shows how to manage it. So does our old work; see AGARD Lecture Series #61, Paper 3.3, especially the distinction between D^f and D^c in Eqs. 18 through 27.

Equation 15 of the <u>Applied Optics</u> paper shows how easy it is to extract inherent properties of the medium from spherical moments $L_n(z)$ of the radiance. For the planar case, let $\dot{L}$ denote dL/dz,

$$\left.\begin{aligned} A_0 &= -\dot{L}_1/L_0 \\ A_1 &= -(\dot{L}_0 + 2\dot{L}_2)/3L_1 \\ A_2 &= -(2\dot{L}_1 + 3\dot{L}_3)/5L_2 \end{aligned}\right\} \qquad (2)$$

Here the A_n are an alternate description of the desired inherent properties A_0 is just <u>a</u>, the absorption constant, and in general

$$A_n = \alpha - S_n \qquad S_n = \int \sigma(\psi)\ P_n(\cos\psi)d\omega, \qquad (3)$$

the S_n being spherical moments of σ, the volume scattering function. (With several of these moments, one could reconstitute $\sigma(\psi)$.) In the spherical case, $dL/dz \rightarrow dL/dr$, and each of Eqs. 2 has terms in L/r as well as dL/dr.

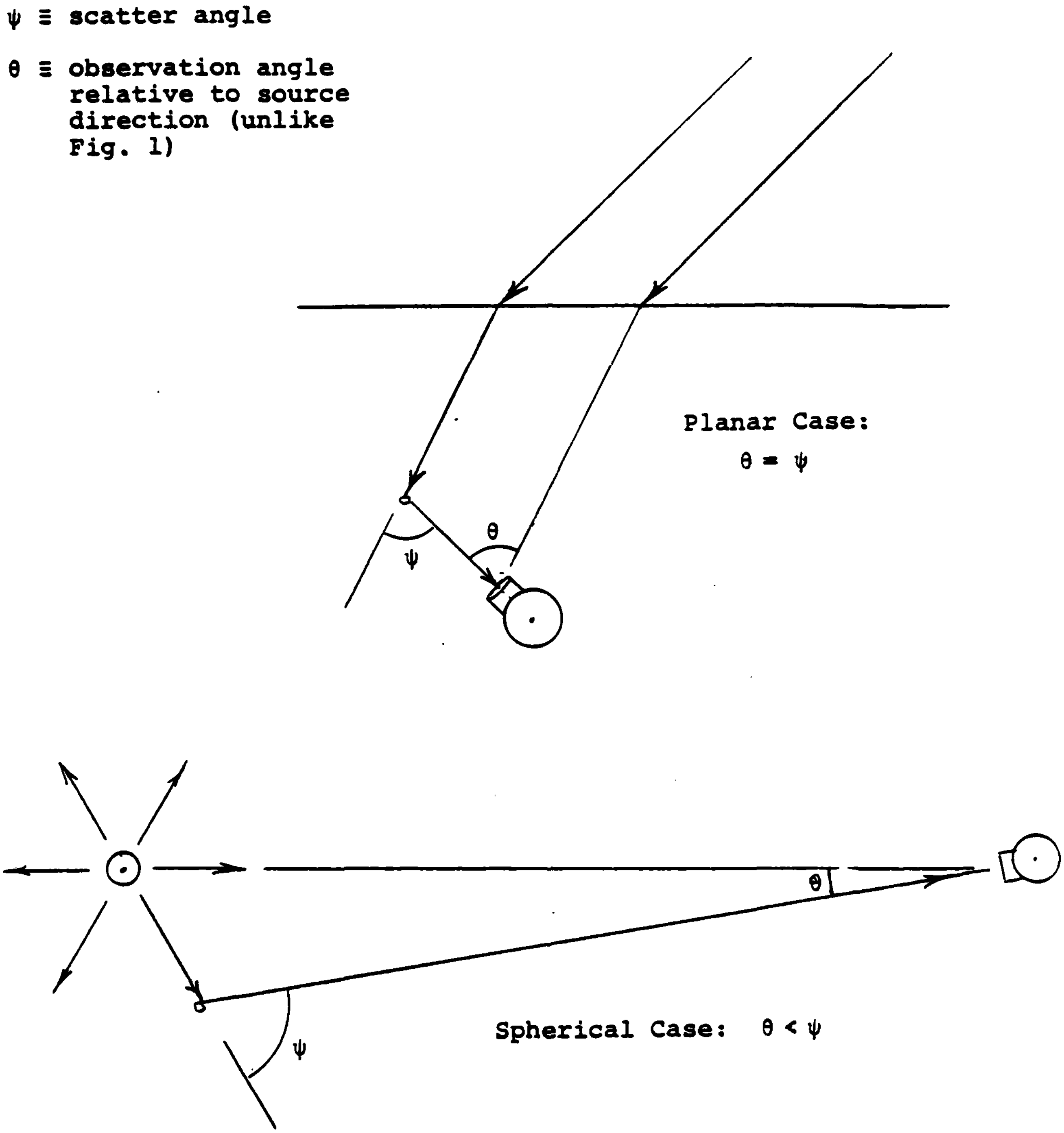

Fig. 3. Effect of Spherical Geometry, which Tends to Collimate the Radiance

We have made an argument that the A_n series, Eq. 3, is a preferable way to specify the inherent properties of the medium. As described in Appendix A of the paper in Applied Optics, the A_n avoid certain problems in resolution of the instruments, and they extend to cases of infinite α.

We must be careful to give credit where due. Optical oceanographers have traditionally measured "scalar irradiance", which is just L_0, and "up- and downwelling vector irradiance", which are hemispherical halves of L_1. They have recognized that these are sufficient to determine A_0, the absorption, using the first of Eqs. 2, which is called Gershun's law. But there most of them stop, apparently unaware that this is only the first of a long series of useful equations in spherical moments. The exception is Zaneveld who has noted the series in Eq. 2, but did not design a meter around it.

Eq. 2 shows the utility of simple meters that measure L_n, and this is the main thrust of the paper in Applied Optics. Section V describes a set of matched meters each of which has a mirror of computed shape, a figure of revolution that can be turned on a lathe (or replicated in plastic from one that has been turned). The shapes are computed so that the angular response function of the meter is a polynomial in $\cos\theta$. (These polynomials are linear combinations of the axi-symmetric spherical harmonics, which are polynomials in $\cos\theta$.) These response functions let the infinite series of spherical harmonics be terminated so that one can isolate the small number to be used (in Eqs. 3) to find inherent properties of the medium.

3.0 SUMMARY OF PUBLICATIONS

Since one objective of this contract (N00014-81-C-0288) was to publish in the open literature, we present here a summary of those publications that resulted from this contract alone. (The preceeding technical discussion mentioned publications done under other contracts as well.)

The first of these is the one called WS for short. It is a "magnum opus" (35 pp.) equivalent to two normal publications, one on mathematical theorems and another on numerical examples.

Published Papers

1. W.H. Wells, J.J. Sidorowich, "Anisotropic Multiple Scattering of Collimated Irradiance", Annals of Phys. 144, pp. 203-237 (Dec 82).

2. W.H. Wells, "Techniques for Measuring Radiance in the Sea and Air", Appl. Opt. 22, pp. 2313-2321 (Aug 83).

Submitted to Journals

3. J.J. Sidorowich and W.H. Wells, "Multiple Scattering in Stratified Inhomogeneous Media", J. Computational Phys. (Paper has been accepted by journal.).

4. W.H. Wells, John J. Sidorowich, "Computational Techniques for Radiative Transfer by Spherical Harmonics", J. Quant. Spec. and Rad. Transfer (manuscript in the mail, preprints sent to ONR).

Unpublished Report

5. "Errors in Optical Oceanographic Measurements Caused by the Hull and Shadow of the Floating Platform", memo to T. Dickey, M. Blizard (1982).

Symposia

Our various results have been described at two scheduled symposia:

- AGARD "Propagation Factors Affecting Remote Sensing by Radio Waves", Oberammergau, Bavaria, May 83.
- Army CSL Conference, Edgewood, MD, June 83.

Seminars

Two scheduled seminars at Johns Hopkins APL, June 83.

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